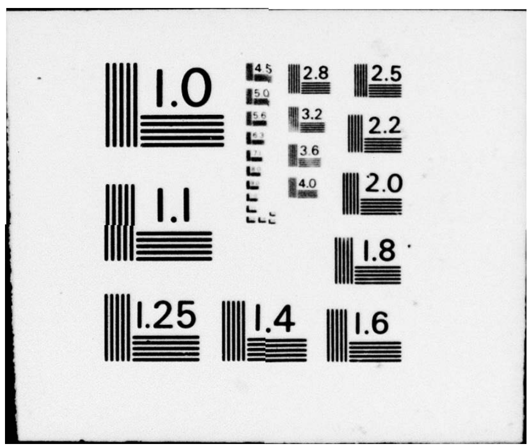
AD-A033 413 GENERAL ELECTRIC CO SCHENECTADY N Y AUTOMATION AND C--ETC F/G 9/4
DYNAMIC BANDWIDTH.(U)
OCT 76 T G KINCAID N00014-71-C-0229
UNCLASSIFIED SRD-76-109 NL

1 OF 1
ADA033413

END
DATE
FILMED
2 - 77


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6

NR 187-401
Cod 220

2

DYNAMIC BANDWIDTH

TECHNICAL REPORT TO OFFICE OF NAVAL RESEARCH

Contract No. N00014-71-C-0229

Period Covered: January 1975 to June 1975

T.G. Kincaid
Automation and Control Lab~~oratory~~
Corporate Research and Development
GENERAL ELECTRIC CO~~MPANY~~
Schenectady, ~~New York~~ 12301
N.Y.

409967

October 1976

SRD-76-109

(6) DYNAMIC BANDWIDTH.

(12) 18p.

TECHNICAL REPORT TO OFFICE OF NAVAL RESEARCH

Contract No. N00014-71-C-0229

(15)

(14) SRD-76-219

(9) Technical rept.

Period Covered: Jan[redacted]to Jun[redacted]975

(10) T.G. Kincaid
Automation and Control Laboratory
Corporate Research and Development
GENERAL ELECTRIC COMPANY
Schenectady, New York 12301

(11) Oct[redacted]76

409 967

ABSTRACT

Rihaczek has defined the dynamic bandwidth of a signal as the square root of the magnitude of the derivative of the instantaneous frequency. This tutorial report reviews the work of Rihaczek and Baghdady on the practical applications of this quantity. It is shown that the dynamic bandwidth is useful as a measure of 1) the frequency spread of the signal energy about the instantaneous frequency, 2) the time stability of the instantaneous frequency, and 3) the minimum bandwidth of a filter that can pass the signal without significant phase distortion.

A

GENERAL ELECTRIC

TABLE OF CONTENTS

B

DYNAMIC BANDWIDTH

T. G. Kincaid

1. INTRODUCTION

The purpose of this report is to bring attention to the measure of "dynamic bandwidth" defined by Rihaczek[1] and to urge its use as an aid in signal modeling and system design.

Consider a complex signal,

$$x(t) = a(t)\exp[j\theta(t)], \tag{1.1}$$

where the amplitude a(t) and the phase θ(t) are real. The well-known definition of the instantaneous frequency of x(t) is

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \tag{1.2}$$

where $f_i(t)$ is in hertz when t is in seconds. Less well known is the dynamic bandwidth of x(t) defined by

$$b(t) = \left|\frac{df_i(t)}{dt}\right|^{1/2} \tag{1.3}$$

where b(t) is in Hertz when t is in seconds. Note that the dynamic bandwidth is related to the phase by

$$b(t) = \left|\frac{1}{2\pi}\frac{d^2\theta}{dt^2}\right|^{1/2} \tag{1.4}$$

The rationale for the definition of dynamic bandwidth is contained in the work of Rihaczek,[1] and the definition is further justified by the earlier work of Baghdady.[2, 3] The results obtained by Rihaczek (discussed in Section 2) show that the dynamic bandwidth has meaning at each time point in the sense of the signal energy being concentrated in a range of frequencies about the instantaneous frequency. He also shows that the reciprocal of the dynamic bandwidth is a measure of the time over which the instantaneous frequency is essentially constant. The work of Baghdady (discussed in Section 3) relates the dynamic bandwidth to the narrowest filter bandwidth that can pass the signal without significant phase distortion.

A word about what the dynamic bandwidth does not measure is advisable. First, since the dynamic bandwidth takes only frequency fluctuations into account, it does not include any amplitude modulation contributions to the signal bandwidth. It is, therefore, most useful for signals with slow amplitude modulation (compared to the frequency modulation). Second, the dynamic bandwidth is not a measure of the overall signal bandwidth. It only measures

the frequency spread at a particular instant. Large slow swings in instantaneous frequency can result in an overall signal bandwidth much larger than the maximum dynamic bandwidth. Third, when the dynamic bandwidth is used to specify the bandwidth of a low phase distortion filter, the low phase distortion occurs at all frequencies, but significant amplitude distortion can occur unless the signal is confined to the filter passband.

In summary, the results described in this report show that the dynamic bandwidth gives the following information at each time point:

1. The frequency spread of the signal energy about the instantaneous frequency.
2. The time interval over which the instantaneous frequency is stable.
3. The minimum bandwidth of a filter which can pass the signal without significant phase distortion.

2. SIGNAL ENERGY DISTRIBUTION

Many authors [see references in Rihaczek[(1)]] have described signal energy as simultaneously distributed in time and frequency. The work of Rihaczek collects the ideas of the previous authors into a unified theory. For a complex signal x(t), with Fourier transform X(f), he summarizes the previous descriptions by defining the complex energy density in time and frequency as

$$e(t, f) = x(t)\, X(f)^{*} \exp[-j2\pi f t] \qquad (2.1)$$

He justifies this definition by showing that 1/2 the integral of e(t, f) within a time interval T and frequency band B does indeed give the signal energy within this time-frequency cell.

Rihaczek's investigation of the interesting properties of the complex energy density function is recommended reading, but contains more information than is necessary here. For the purposes of this discussion, only his use of the function to examine the distribution of energy in time and frequency is important. To begin this investigation, he first writes the signal and its Fourier transform in terms of their amplitude and phase; i.e.,

$$x(t) = |x(t)| \exp[j\theta(t)] \qquad (2.2)$$

$$X(f) = |X(f)| \exp[j\phi(f)] \qquad (2.3)$$

If these are substituted into Eq. (2.1), then the signal energy E is shown in (1) to be given by the integral

$$E = 1/2 \iint dt\,df\, |x(t)||X(f)| \exp[j\theta(t) - j\phi(f) - 2\pi f t] \qquad (2.4)$$

The time and frequency integrals above can each be evaluated by the method of stationary phase (see Appendix A). This method uses the fact that the significant contributions to the integrals occur at the points where the phase of the integral is stationary, i.e., the phase function has zero slope. The method requires that the second derivative of the phase is not zero at these points, and that the amplitude variation is slow compared to the phase variation in the region about the points. The stationary points can be found by setting the time and frequency phase derivatives equal to zero, and solving for the coordinates of the stationary points.

Setting the time derivative of the phase of e(t, f) equal to zero gives the following equation for the times at which the phase is stationary:

$$f = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \quad (\equiv f_i(t)) \tag{2.5}$$

This equation is just the definition of the instantaneous frequency of x(t). For each f, the values of t which solve this equation are the stationary time points corresponding to frequency f. For any physical signal, x(t) has an instantaneous frequency over its entire time extent; thus every time point on the extent of x(t) is a stationary time point for some f.

Setting the frequency derivative of the phase of e(t, f) equal to zero gives the following equation for the frequencies at which the phase is stationary.

$$t = -\frac{1}{2\pi}\frac{d\phi(f)}{df} \quad (\equiv t_g(f)) \tag{2.6}$$

This equation is the definition of the familiar group delay t_g of x(t), the dual of the instantaneous frequency.[4] At each t, the values of f which solve this equation are the stationary frequency points corresponding to that particular value of the time t. For any physical signal, x(t) has a group delay for all frequencies in its band. Therefore, every frequency in the band occupied by x(t) is a stationary point for some t. (Note that for theoretical signals with spectral line components, the group delay is not defined at the line frequencies.)

It has been argued that almost every value of t and f within the time-frequency extent of x(t) are stationary points for their respective integrals. However, a significant contribution to the double integral, and hence to the signal energy, occurs only in the region of the t, f plane where the stationary points are simultaneous solutions to Eqs. (2.5) and (2.6). Each of these equations describes curves in the time frequency plane. Therefore, their simultaneous solutions are at the intersections of these two curves. It is shown in Appendix B that these two curves intersect all along their length, i.e., they are the same curve. Mathematically, this means that $f_i()$ and $t_g()$ are functional inverses. That is,

$$f_i(t_g(f)) = f \tag{2.7}$$

Therefore, either Eq. (2.5) or (2.6) is the locus of all the points in the t, f plane at which e(t, f) is simultaneously stationary in both t and f. It follows

that at each point in time the signal energy is concentrated in an inteval about the instantaneous frequency. The width of this interval in the frequency direction is the band of frequencies over which the phase of e(t, f) is nearly a constant as a function of frequency. Similarly, the width in the time direction is the duration of time over which the phase of e(t, f) is nearly constant as a function of time. It is shown below that these widths are, respectively, the dynamic bandwidth and a quantity called the signal relaxation time.

Rihaczek determines the width of these intervals by noting that the requirement for the phase of e(t, f) to be nearly constant on an interval is that $\theta(t)$ and $\phi(f)$ are nearly linear on the interval. He defines these intervals to be those over which the quadratic terms in the Taylor expansions of $\theta(t)$ and $\phi(f)$ do not deviate from linearity by more than $\pi/4$. He uses this criterion to determine the time interval width by noting that the magnitude of the quadratic term in the Taylor expansion of $\theta(t)$ about the stationary point t_0 is

$$1/2\ (t-t_0)^2 \left|\frac{d^2\theta}{dt^2}\right|_{t_0} .$$

By using this term as an approximation to the deviation from linearity, the interval can be determined as the time between the two points where its magnitude is $\pi/4$. Computing this number, called the signal <u>relaxation time</u> r, gives

$$r = \left|\frac{1}{\frac{1}{2\pi}\frac{d^2\theta(t)}{dt^2}}\right|^{1/2} = \left|\frac{1}{\frac{df_i(t)}{dt}}\right| \tag{2.8}$$

when the derivatives are evaluated at time t_0. Using the same criterion, the frequency interval about the stationary point f_0 over which the phase is nearly constant can be shown to be

$$b = \left|\frac{1}{\frac{1}{2\pi}\frac{d^2\phi(f)}{df^2}}\right|^{1/2} = \left|\frac{1}{\frac{dt_g(f)}{df}}\right| \tag{2.9}$$

when the derivatives are evaluated at f_0. The use of b is deliberate, since this is the dynamic bandwidth defined in Eq. (1.3). This is proved from the fact that f_i and t_g are inverse functions, and hence their derivatives (i.e., their slopes) are reciprocals at the same point on their trajectory. That is,

$$\frac{df_i(t)}{dt} = \frac{1}{\frac{dt_g(f)}{df}} \tag{2.10}$$

when evaluated at the same point on the projectory of $f_i(t)$ and $t_g(f)$. This relationship is pictured in Figure 2.1.

Substitution of Eq. (2.10) into Eq. (2.9) gives

$$b = \left|\frac{df_i(t)}{dt}\right|^{1/2} = \left|\frac{1}{2\pi}\frac{d^2\theta(t)}{dt^2}\right|^{1/2} \tag{2.11}$$

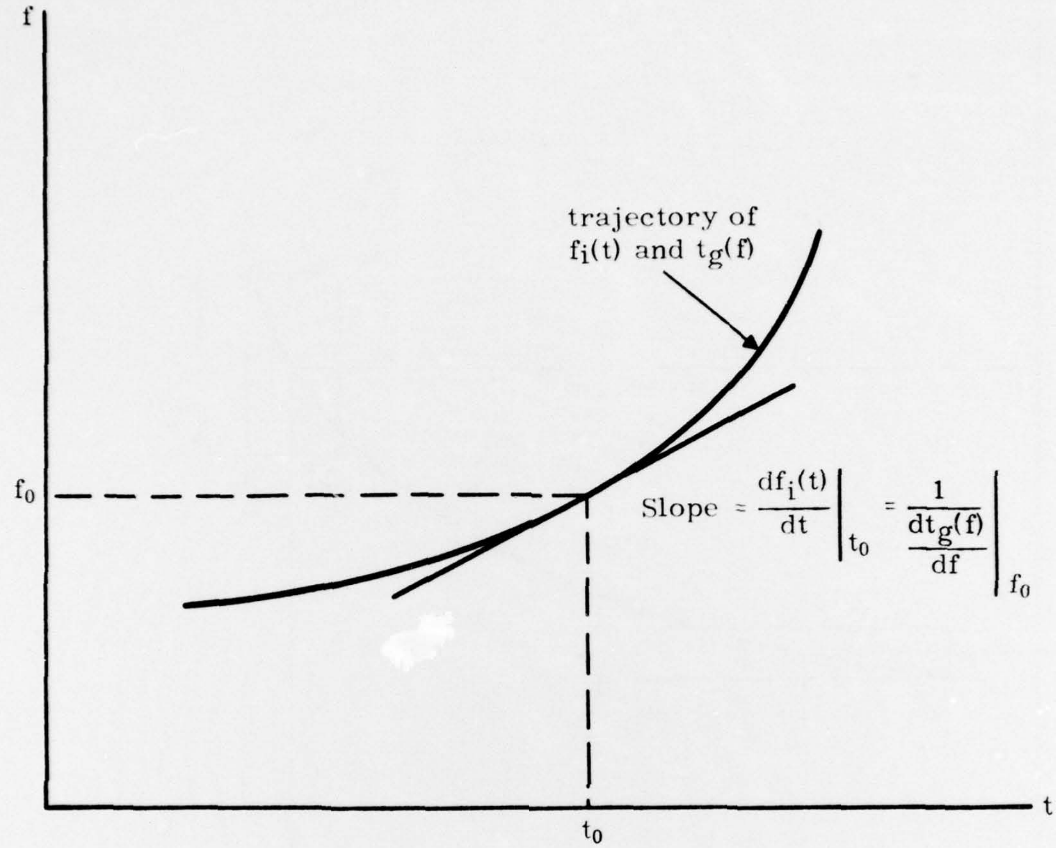


Figure 2.1. The slope of the instantaneous frequency and group delay trajectory at the point t_0, f_0 in the time-frequency plane.

which is the definition of dynamic bandwidth given in Eq. (1.3). A comparison of Eqs. (2.8) and (2.11) shows that b and r are reciprocals when computed at the same point on the $f_i(t)$ and $t_g(f)$ trajectory. This leads to the time-frequency uncertainty relation derived by Rihaczek,

$$br = 1. \tag{2.12}$$

This result shows that at each instant of time the signal energy is concentrated in a unit area about the instantaneous frequency trajectory in the time-frequency plane. This area changes in shape as time progresses. Slow instantaneous frequency changes spread the energy more in time, while rapid changes spread it more in frequency. These concepts are pictured in Figure 2.2.

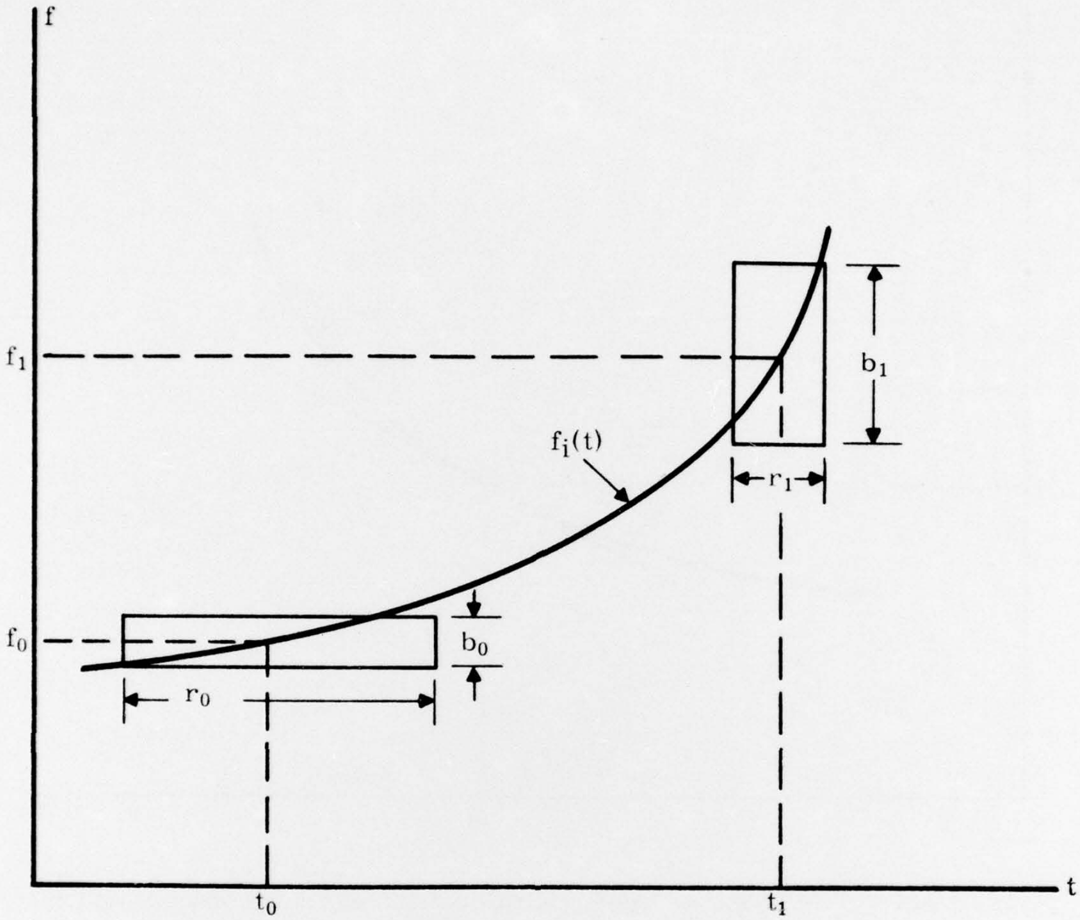


Figure 2.2. Signal energy concentration about the instantaneous frequency trajectory at two points in the time frequency plane.

3. FILTER BANDWIDTH

Baghdady[2, 3] has determined the minimum filter bandwidth consistent with low distortion filtering of a frequency (i. e., phase) modulated signal. He determines this by considering a system in which a pure phase modulated signal, $x(t) = \exp[j\theta(t)]$, is the input to a filter with radian frequency response $H(\omega)$. The filter output $y(t)$ has "low distortion" if at each time instant $y(t)$ is approximately the response of the filter to a single frequency input at the instantaneous frequency of $x(t)$. That is, the filter has "low distortion" if

$$y(t) \simeq H(\theta'(t)) \exp[j\theta(t)] \qquad (3.1)$$

where the prime denotes derivative. Baghdady shows that the actual response is given by

$$y(t) = H(\theta'(t))\,[1+\varepsilon(t)]\,\exp[j\theta(t)] \tag{3.2}$$

where $\varepsilon(t)$ is a relative error term which satisfies the condition

$$|\varepsilon(t)| \leq 1/2\,|\theta''(t)|_{max}\left|\frac{H''(\omega)}{H(\omega)}\right|_{max} \tag{3.3}$$

For low distortion, the right side of the above equation must be << 1; i.e., the relative error must be small.

The ratio $|H''(\omega)/H(\omega)|$ is called the sluggishness ratio. It is a measure of the ability of the energy storage elements in the filter to respond to changes in the instantaneous frequency of the signal. Baghdady has calculated this ratio for Butterworth filters up to sixth order,[2, 3] and for cascaded first- and second-order Butterworth filters up to six sections.[3] In all cases, the maximum value of the sluggishness ratio is inversely proportional to the square of the filter half power bandwidth. This relation appears to be generally true, and the fact that it is true for many filters of interest allows the maximum value of the ratio for these filters to be written

$$\left|\frac{H''(\omega)}{H(\omega)}\right|_{max} = \frac{k}{(2\pi B)^2} \tag{3.4}$$

The bandwidth B is in Hertz, and the proportionately constant k is called the stiffness index of the filter.

Baghdady gives the value of k for the filters mentioned above.[3] For a first-order Butterworth filter (i.e., a single tuned resonant circuit) k = 8. The index increases for higher order Butterworth filters. A simple calculation shows that for a sinc function filter $k \approx \pi^2$ if the bandwidth is defined as the distance from the peak of the spectrum to the first zero. An interesting case is the filter with an exponentially decaying frequency response $\exp[-|f|/B]$, for which k = 1 if the bandwidth is defined as the distance from the peak to the frequency where the amplitude is down by 1/e.

Substituting Eq. (3.4) and the definition of instantaneous frequency [Eq. (1.4)] into Eq. (3.3) gives an alternate expression for the relative error bound.

$$|\varepsilon(t)| \leq \frac{k}{4\pi}\,\frac{b^2 max}{B^2} \tag{3.5}$$

For low distortion reproduction of the phase modulation at the filter output, the right side of the above inequality should be on the order of 1/10. For convenience, make it $1/4\pi$. Then the minimum bandwidth of the low distortion filter is given by

$$B = \sqrt{k}\, b_{max} \tag{3.6}$$

This equation shows that the filter bandwidth is proportional to the signal dynamic bandwidth, as it should be for a meaningful definition of dynamic

bandwidth. The constant of proportionality depends upon the type of filter through its stiffness index k. For a first-order Butterworth filter, $k = 8$ and $B = \sqrt{2}\, b_{max}$. For a sinc filter, $k = \pi^2$ and $B = \pi\, b_{max}$. For the exponential filter mentioned above, $k = 1$ and $B = b_{max}$.

A word about the practical uses of Eq. (3.5) to specify filter bandwidth. This design equation was derived to meet a low phase distortion criterion. The low phase distortion will hold at all frequencies, even outside the filter pass band. However, the amplitude of the signal, in general, will be distorted outside the pass band. If Eq. (3.5) is used to design a bank of filters-- say, for spectrum analysis--the criterion given by Eq. (3.5) is ideal for determining the minimum bandwidth, since the signal will always be in one of the filters. However, for a tuning or noise rejection filter, care must be taken that the signal remains in the pass band to prevent amplitude distortion.

4. EXAMPLES

4.1 Single Frequency

$$x(t) = \exp[j2\pi f_0 t]$$

Phase: $\theta(t) = 2\pi f_0 t$

Instantaneous Frequency: $f_i(t) = f_0$

Dynamic Bandwidth: $b(t) = 0$

The zero dynamic bandwidth is consistent with the notion of zero bandwidth for a single frequency signal. Note that the group delay and relaxation time are not defined for this signal. The instantaneous frequency trajectory of this signal is just a straight line parallel to the time axis, as shown in Figure 4.1.

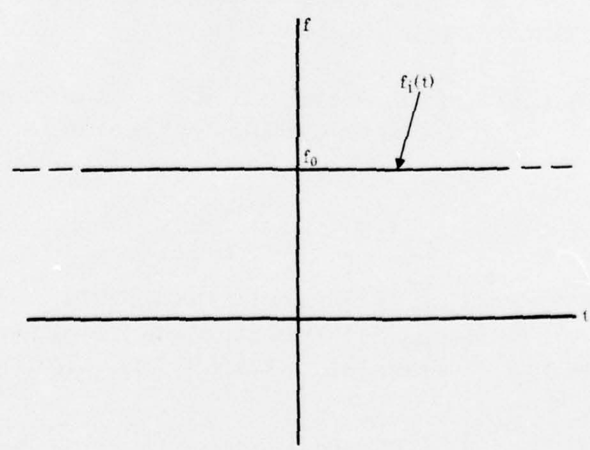


Figure 4.1. The instantaneous frequency trajectory for a single frequency signal.

4.2 FM Slide

$$x(t) = \exp[j\pi\alpha t^2]$$

Phase: $\theta(t) = \pi\alpha t^2$

Instantaneous Frequency: $f_i(t) = \alpha t$

Dynamic Bandwidth: $b(t) = |\alpha|^{1/2}$

The instantaneous frequency of the FM slide has a linear time-frequency trajectory and a constant dynamic bandwidth. The group delay is f/α and the relaxation time is $|\alpha|^{-1/2}$. The instantaneous frequency trajectory is shown in Figure 4.2, along with the frequency distribution of energy given by the dynamic bandwidth.

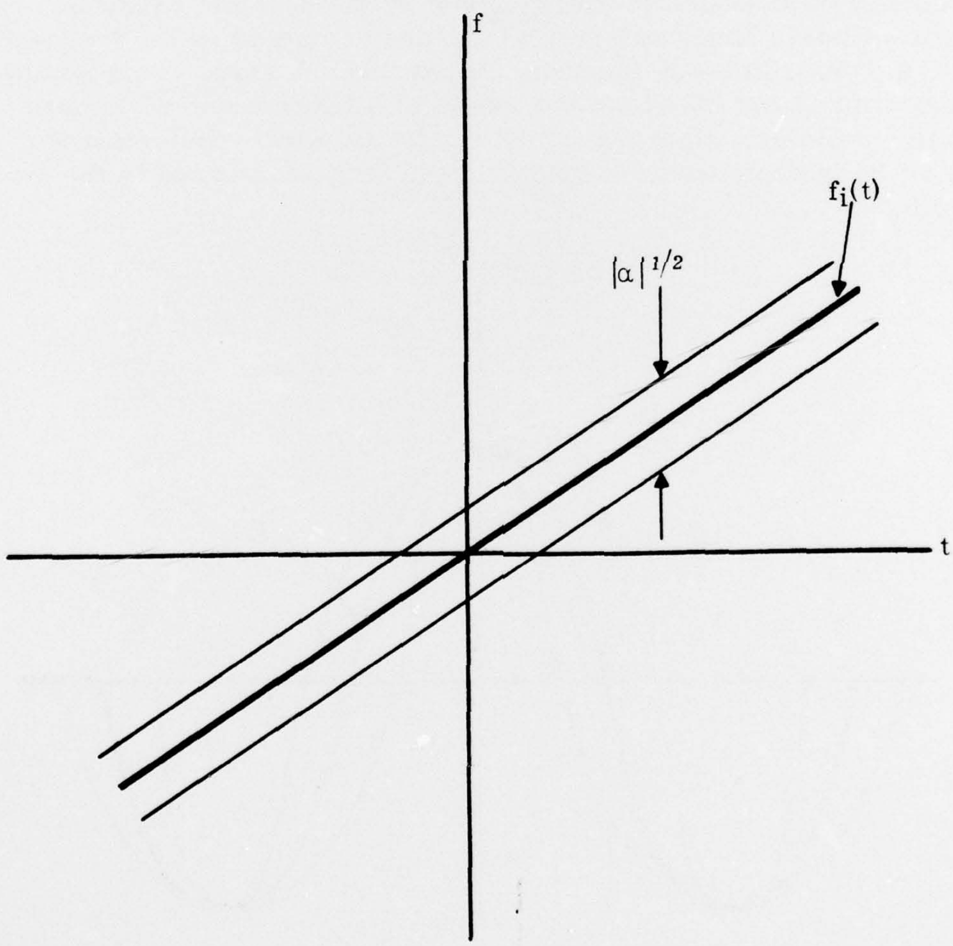


Figure 4.2. The frequency distribution of signal energy about the instantaneous frequency trajectory of an FM slide.

4.3 Sinusoidal Phase Modulation

$$x(t) = \exp[j\beta\sin 2\pi f_0 t]$$

Phase: $\theta(t) = \beta\sin 2\pi f_0 t$

Instantaneous Frequency: $f_i(t) = \beta f_0 \cos 2\pi f_0 t$

Dynamic Bandwidth: $b(t) = f_0 |2\pi\beta\sin 2\pi f_0 t|^{1/2}$

When the phase modulation is sine wave the instantaneous frequency is a cosine of the same frequency, and the dynamic bandwidth oscillates at twice the modulation frequency. Since the signal x(t) is periodic, it has a line spectrum, and the group delay and relaxation time are not defined.

The instantaneous frequency trajectory is shown in Figure 4.3, along with the frequency distribution of energy given by the dynamic bandwidth. The maximum dynamic bandwidth is $\sqrt{2\pi\beta}\, f_0$, and occurs when the frequency modulation (i.e., instantaneous frequency) goes through zero. This maximum value determines the minimum bandwidth of a filter required to pass x(t) without phase distortion. Note that there can be amplitude distortion if the amplitude of the instantaneous frequency βf_0 is large compared to the dynamic bandwidth.

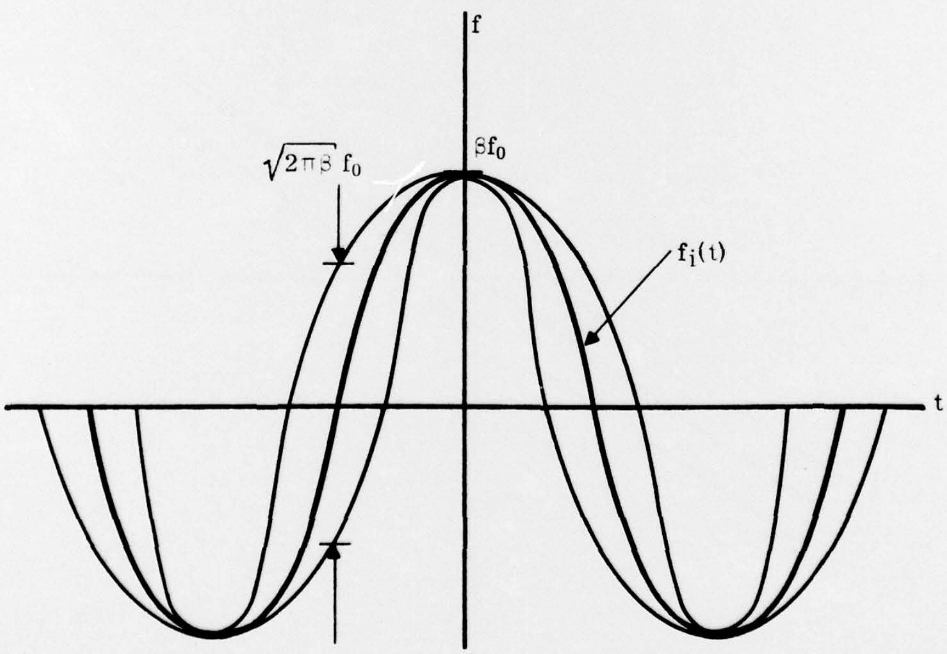


Figure 4.3. The frequency distribution of signal energy about the instantaneous frequency trajectory of a sinusoidally phase modulated signal.

REFERENCES

1. A.W. Rihaczek, "Signal Energy Distribution in Time and Frequency," IEEE Trans. IT-14, No. 3, 369-374 (May 1968).

2. E.J. Baghdady, "Theory of Low-Distortion Reproduction of FM Signals in Linear Systems," IRE Trans. CT-5, 202-214 (Sept. 1958).

3. E.J. Baghdady, "Analog Modulation Systems," Lectures on Communication System Theory, E.J. Baghdady, ed., McGraw-Hill, New York (1961), Chap. 19.

4. A. Papoulis, The Fourier Integral and Its Applications, McGraw-Hill, New York (1962), p. 135.

5. A. Papoulis, Ibid., pp. 139-143.

APPENDIX A

This Appendix is a summary of the method of stationary phase as given by Papoulis.(5)

Consider the integral

$$I = \int_{-\infty}^{\infty} a(t) \exp[j\alpha(t)] \tag{A1}$$

If $\alpha(t)$ varies rapidly compared to $a(t)$, then I can be evaluated by the method of stationary phase. Assume $\alpha'(t) = 0$ at only one point t_0, called the stationary point of $\alpha(t)$. Then

$$I \simeq a(t_0)\sqrt{\frac{2\pi}{|\alpha''(t_0)|}} \exp[j\alpha(t_0) \pm \pi/4] \tag{A2}$$

where the sign in front of $\pi/4$ is the same as the sign of $\alpha''(t_0)$.

Suppose that $\alpha(t)$ has n stationary points, $t_1, t_2, \ldots, t_n$. Then the method of stationary phase gives

$$I = \sum_{i=1}^{n} a(t_i)\sqrt{\frac{2\pi}{|\alpha''(t_i)|}} \exp[j\alpha(t_i) \pm \pi/4] \tag{A3}$$

where the sign of $\pi/4$ is determined as above.

APPENDIX B

Consider the signal

$$x(t) = |x(t)| \exp[j\theta(t)] \tag{B1}$$

on the interval T. The instantaneous frequency of x(t) is the function

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \tag{B2}$$

Assume the interval T is chosen so that no value of the instantaneous frequency occurs more than once. Mathematically, this means that

$$\frac{df_i(t)}{dt} = \frac{d^2\theta(t)}{dt^2} \neq 0 \tag{B3}$$

on the interval T.

The Fourier transform of x(t) on T is given by

$$X(f) = \int_T dt\,|x(t)|[\exp j\theta(t) - j2\pi ft]$$

$$= |X(f)| \exp[j\phi(f)] \tag{B4}$$

The group delay of x(t) on T is the function

$$t_g(f) = -\frac{1}{2\pi} \frac{d\phi(f)}{df} \tag{B5}$$

The instantaneous frequency is a function of time, and the group delay is a function of frequency. Each of these functions describes a trajectory in the time-frequency plane. It is shown below that these two trajectories are the same; i.e., $f_i(\ )$ and $t_g(\ )$ are inverse functions.

The proof of this is as follows. For any value of f, the stationary point of the signal phase is the time point t_s which solves the equation

$$f = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \tag{B6}$$

Choose f so that t_s is not near the edge of the interval. Then, by the principal of stationary phase, the Fourier transform X(f) is closely approximated by

$$X(f) \simeq |x(t)| \sqrt{\frac{2\pi}{|\theta''(t)|}} \exp[j\theta(t) - j2\pi ft \pm \pi/4]\Big|_{t_s} \tag{B7}$$

where the sign of $\pi/4$ is chosen the same as the sign of $\theta''(t_s)$. Thus, the phase of X(f) is

$$\phi(f) = \theta(t_s) - 2\pi t_s f \pm \pi/4 \tag{B8}$$

Note that t_s is a function of f. By definition

$$t_g(f) = -\frac{1}{2\pi}\frac{d\phi(f)}{df}$$

$$= -\frac{1}{2\pi}\frac{d\theta(t_s)}{df} + t_s + \frac{dt_s}{df} f$$

$$= -\frac{1}{2\pi}\frac{d\theta(t_s)}{dt_s}\frac{dt_s}{df} + t_s + \frac{dt_s}{df} f \qquad \text{(B9)}$$

However, since t_s is the solution to Eq. (B2) for $f_i(t_s) = f$; therefore,

$$\frac{d\theta(t_s)}{dt_s} = 2\pi f \qquad \text{(B10)}$$

Substitution of (B9) into (B8) gives

$$t_g(f) = t_s(f) \qquad \text{(B11)}$$

That is, the stationary time point for any f is the group delay time for that f. Therefore, $t_g(f)$ is a solution to Eq. (A2), and

$$f_i(t_g(f)) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}\bigg|_{t_g(f)} = f \qquad \text{(B12)}$$

which was to be proved.

DISTRIBUTION LIST

Office of Naval Research Department of the Navy Arlington, Virginia 22217 Attn: Code 220 Code 222	2 copies
Director Naval Research Laboratory Technical Information Division Washington, D.C. 20375 Attn: Dr. John Munson	1 copy
Commander Naval Undersea Center San Diego, California 92132 Attn: Mr. G.B. Anderson Dr. H.A. Schenck	1 copy
Officer in Charge Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320	1 copy
Commander Naval Air Development Center Warminster, Pennsylvania 18974	1 copy
Officer in Charge Naval Ship Research and Development Center Carderock Laboratory Bethesda, Maryland 20034 Attn: Mr. C. Olson	1 copy
Commanding Officer Naval Intelligence Support Center 4301 Suitland Road Washington, D.C. 20390 Attn: STIC 34 Mr. E. Bissett	2 copies
Officer in Charge Naval Ship Research and Development Center Annapolis Laboratory Annapolis, Maryland 21402	1 copy

Commander
Naval Sea Systems Command
Naval Sea Systems Command Headquarters
Washington, D.C. 20362
Attn: CAPT R. Bruce Gilcrist (Code 06H2)

1 copy

Commander
Naval Air Systems Command
Naval Air Systems Command Headquarters
Washington, D.C. 20361
Attn: Code 533

1 copy

Commander
Naval Electronic Systems Command
Naval Electronic Systems Command Headquarters
Washington, D.C. 20360
Attn: CAPT J.C. Bajus, Code 03
CDR H. Miller, Code 320
CAPT D. Jackson (PME 124)

3 copies

Commander
Oceanographic System, Atlantic
Box 100
Norfolk, Virginia 23511

1 copy

Commander
Oceanographic System, Pacific
Box 1390
FPO San Francisco 96610

1 copy

Chief of Naval Operations
Department of the Navy
Washington, D.C. 20350
Attn: OP-951F
CAPT A.H. Gilmore

2 copies

Director of Navy Laboratories
Room 1062, Crystal Plaza Bldg. 5
Department of the Navy
Washington, D.C. 20390
Attn: Mr. James Probus

1 copy

Office of the Assistant Secretary of the Navy
Department of the Navy
Room 4D745, Pentagon
Washington, D.C. 20350
Attn: Mr. Harry Sonnemann

1 copy

Office of the Secretary of Defense DDR&E Room 3E1044, Pentagon Washington, D.C. 20350 Attn: Mr. David R. Heebner	1 copy
Office of the Secretary of Defense ODDR&E Room 3D1048, Pentagon Washington, D.C. 20350 Attn: Mr. Stanley A. Peterson Mr. Gerald Cann	2 copies
Mr. A.J. Tachmindji Advanced Research Project Agency 1400 Wilson Boulevard Arlington, Virginia 22209 Attn: Doc. Control Rm 633	1 copy
Bell Telephone Laboratory Whippany Road Whippany, New Jersey 07981 Attn: Dr. Bruce Bogart Dr. Peter Hirsch	2 copies
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2 copies
Institute for Acoustical Research Miami Division of the Palisades Geophysical Institute 615 S.W. 2nd Avenue Miami, Florida 33130 Attn: Mr. M. Kronengold Dr. J. Clark Dr. S. Adams Dr. B. Jobst	1 copy
General Electric Company P.O. Box 8 Schenectady, New York 12301 Attn: Dr. Charles Stutt Dr. Thomas Kincaid Dr. Henry Scudder III	1 copy

General Electric Company Heavy Military Electronic Systems P.O. Box 1122 Syracuse, New York 13201 Attn: Mr. Don Winfield	1 copy
Texas Instruments Inc. P.O. Box 5474 13500 North Central Expressway Dallas, Texas 75222 Attn: Dr. Thomas W. Ellis	1 copy
Jet Propulsion Laboratory 4800 Oak Grove Drive Pasadena, California 91103 Attn: Mr. Carl Theile	1 copy
University of Michigan Cooley Electronics Laboratory Box 622 Ann Arbor, Michigan 48105 Attn: Dr. T.G. Birdsall Mr. K. Metzgar	1 copy
Systems Control Inc. 1801 Page Mill Road Palo Alto, California 94304 Attn: Dr. L. Seidman	1 copy